Closing tonight: 3.1-2
Closing Mon: 3.3 (finish sooner!)
Closing next Fri: 3.4 (part 1)

Exam 1 is Tuesday, Jan 31st in your normal quiz section.
Covers 2.1-2.3,2.5-2.8, 3.1-3.3.

- One 8.5 by 11 inch sheet of ***handwritten*** notes (front and back)
- A Ti-30x IIs calculator (this model only!)
- Pen or pencil (no red or green)
- No make-up exams.

All homework is fair game. Know the concepts well. Practice on old exams.

Entry Task: Find the derivative of

$$y = \frac{2x^2 + 1}{x^3 e^x}$$

3.3 Derivatives of Trig Functions

First a review: you will need to know all the following well in Math 124/5/6.

Triangle definitions

$\sin(x) = \dfrac{\text{opp}}{\text{hyp}}$	$\cos(x) = \dfrac{\text{adj}}{\text{hyp}}$
$\tan(x) = \dfrac{\text{opp}}{\text{adj}}$	$\cot(x) = \dfrac{\text{adj}}{\text{opp}}$
$\sec(x) = \dfrac{\text{hyp}}{\text{adj}}$	$\csc(x) = \dfrac{\text{hyp}}{\text{opp}}$

Thus,

$\sec(x) = \dfrac{1}{\cos(x)}$	$\csc(x) = \dfrac{1}{\sin(x)}$
$\tan(x) = \dfrac{\sin(x)}{\cos(x)}$	$\cot(\text{x}) = \dfrac{\cos(x)}{\sin(x)}$

Know what their graphs look like.

Know their inverses.

Know the standard values (unit circle).

Examples:

$\cos\left(\frac{\pi}{6}\right) =$

$\sec\left(-\frac{\pi}{4}\right) =$

$\tan\left(\frac{2\pi}{3}\right) =$

Know main identities.

$$\sin^2(x) + \cos^2(x) = 1$$

$$2\sin(x)\cos(x) = \sin(2x)$$

For today we need the sum identities:

$\sin(a+b) = \sin(a)\cos(b) + \cos(\text{a})\sin(b)$

$\cos(a+b) = \cos(\text{a})\cos(b) - \sin(a)\sin(b)$

Consider $f(x) = \sin(x)$.

Then

$$\begin{aligned} f'(x) &= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \to 0} \frac{\sin(x+h) - \sin(x)}{h} \end{aligned}$$

Summary:

$\frac{d}{dx}(\sin(x)) = \cos(x)$	$\frac{d}{dx}(\cos(x)) = -\sin(x)$
$\frac{d}{dx}(\tan(x)) = \sec^2(x)$	$\frac{d}{dx}(\cot(x)) = -\csc^2(x)$
$\frac{d}{dx}(\sec(x)) = \sec(\mathrm{x})\tan(\mathrm{x})$	$\frac{d}{dx}(\csc(x)) = -\csc(\mathrm{x})\cot(\mathrm{x})$

3.4 Chain Rule

The **composition** of two function is defined by

$$(f \circ g)(x) = f(g(x))$$

Example:

If f(x) = sin(x), g(x) = x^3, then

$$(f \circ g)(x) = \sin(x^3).$$

Chain Rule:

$$\frac{d}{dx} f(g(x)) = f'(g(x))g'(x)$$

Also written as: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

Example:

$$\frac{d}{dx}\sin(x^3) = \cos(x^3)\, 3x^2$$

Here is a brief "proof sketch" for the chain rule:

From the definition of derivative

$$
\begin{aligned}
\frac{d}{dx} f\big(g(x)\big) &= \lim_{h \to 0} \frac{f(g(x+h)) - f(g(x))}{h} \\
&= \lim_{h \to 0} \left(\frac{f(g(x+h)) - f(g(x))}{g(x+h) - g(x)} \right) \left(\frac{g(x+h) - g(x)}{h} \right) \\
&= f'\big(g(x)\big) g'(x)
\end{aligned}
$$

Examples: Find the derivative

1. $y = (2x^2 + 1)^2$

2. $y = e^{\sin(2x)}$

3. $y = \tan(3x + \cos(4x))$

4. $y = \sin^4(x)$

5. $y = \sin(x^4)$

Identify the “first” rule you would use to differentiate these functions:
(sum, product, quotient or chain?)

1. $y = \sqrt{\sin(x) + x^2 + 1}$

2. $y = \frac{x^4}{\sin(5x+1)}$

3. $y = \sqrt[3]{4x+1}\cos(\sin(2x))$

4. $y = e^{\tan(x)} - 5(x^8+1)^{50}$

5. $y = \left(\frac{x^2-1}{x^4+1}\right)^{10}$